DYNAMICS OF NEUTRAL SHEETS IN A PLASMA

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The change in magnetic field topology as a result of reconnection of field lines plays an important role in processes taking place in the tail of the earth's magnetosphere [1] and laboratory experiments on plasma heating [2]. A number of authors [3-5] have published results on the numerical solution of the MHD plane problem of flow of a plasma containing a neutral sheet. The problem of the development, in a collisionless approximation, of the tearing instability responsible for reconnection was studied in [6].

In the present article we discuss the results of a numerical solution of the cylindrical problem of the peculiarities of plasma flow near a neutral surface under conditions close to the conditions of laboratory experiments in Θ-pinch installations with inverted magnetic fields [2]. The solution is obtained in the approximation of one-fluid magnetohydrodynamics and of the dissipative mechanisms, and only the finite conductivity of the plasma is taken into account in accordance with the data of [2]. To describe the plasma we use the system of equations

$$\begin{gathered}\partial\rho/\partial t = -\operatorname{div}(\rho\mathbf{u}),\ \rho(\partial\mathbf{u}/\partial t + (\mathbf{u}\nabla)\mathbf{u}) = -\nabla p + (1/4\pi)[\operatorname{rot}\mathbf{H}\cdot\mathbf{H}],\\ \partial\mathbf{H}/\partial t = \operatorname{rot}([\mathbf{u}\cdot\mathbf{H}] - (c^2/4\pi\sigma)\operatorname{rot}\mathbf{H}),\\ \partial p/\partial t + \mathbf{u}\nabla p = -\gamma p\operatorname{div}\mathbf{u} + (c^2/16\pi^2\sigma)(\operatorname{rot}\mathbf{H})^2,\\ \operatorname{div}\mathbf{H} = 0,\ p = \rho T/m_i,\ \sigma = \rho e^2/m_e m_i\nu,\ \nu = \nu_0 + \nu_1 T^{-3/2},\end{gathered} \tag{1}$$

where ρ, $\mathbf{u}$, p, and T are the plasma density, macroscopic velocity, pressure, and temperature; **H**, magnetic field strength; σ, plasma conductivity; and ν, frequency of collisions between particles, which we represent in the form of the sum of the Coulomb and anomalous frequencies.

Since we are considering the two-dimensional problem $(\partial/\partial\varphi = 0)$, it is convenient to introduce, in place of the magnetic field strength, the vector potential in accordance with the usual definition $\mathbf{H} = \operatorname{rot}\mathbf{A}$, since in this case: a) only one component $A_\varphi \equiv A$ is different from zero; b) lines $rA = \text{const}$ are magnetic field lines with the help of which it is easy to represent the variation of the field topology; c) the conductivity σ appears in the induction equation not behind the differentiation sign but as a coefficient.

Choosing the characteristic quantities R (the radius of the installation), ρ_0 (the plasma density at the axis), and H_0 (the magnetic field strength at the chamber wall) as the scales of length, density, and magnetic field, we write the system of equations (1) in cylindrical coordinates and dimensionless variables:

$$\begin{gathered}\frac{\partial\rho}{\partial t} + \frac{1}{r}\frac{\partial}{\partial r}(r\rho u) + \frac{\partial}{\partial z}(\rho w) = 0,\quad \rho\left(\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial r} + w\frac{\partial u}{\partial z}\right) = -\frac{1}{2}\frac{\partial p}{\partial r} -\\ -\frac{1}{r}\frac{\partial}{\partial r}(rA)\left\{\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial}{\partial r}(rA)\right) + \frac{\partial^2 A}{\partial z^2}\right\},\quad \rho\left(\frac{\partial w}{\partial t} + u\frac{\partial w}{\partial r} + w\frac{\partial w}{\partial z}\right) = -\frac{1}{2}\frac{\partial p}{\partial z} - \frac{\partial A}{\partial z}\left\{\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial}{\partial r}(rA)\right) + \frac{\partial^2 A}{\partial z^2}\right\},\\ \frac{\partial A}{\partial t} + \frac{u}{r}\frac{\partial}{\partial r}(rA) + w\frac{\partial A}{\partial z} = \nu\left\{\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial}{\partial r}(rA)\right) + \frac{\partial^2 A}{\partial z^2}\right\},\quad \frac{\partial p}{\partial t} + u\frac{\partial p}{\partial r} + w\frac{\partial p}{\partial z} -\\ = -\gamma p\left(\frac{1}{r}\frac{\partial}{\partial r}(ru) + \frac{\partial w}{\partial z}\right) + 2(\gamma - 1)\nu\left\{\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial}{\partial r}(rA)\right) + \frac{\partial^2 A}{\partial z^2}\right\}^2,\\ T = pm_i/\rho,\quad \nu = \nu_0 + \nu_1 T^{-3/2},\\ \nu_0, \nu_1 = \text{const}.\end{gathered} \tag{2}$$

Let us consider the following problem. At the initial time a quiescent plasma $(u = w = 0)$ occupies a region $0 \le r \le R$, $0 \le z \le z_1$ and is placed in a magnetic field $\mathbf{H} = \{0, 0, \tanh\alpha(r - r_0)\}$, where the coefficient α determines the width δ of the neutral sheet and r_0 is the position of the null surface of the magnetic field. The corresponding initial values of the potential are determined from equation

$$A(r, z, 0) = \frac{1}{r}\int_0^r r' H_z(r', z, 0)\, dr'.$$

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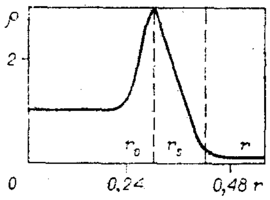

Fig. 1

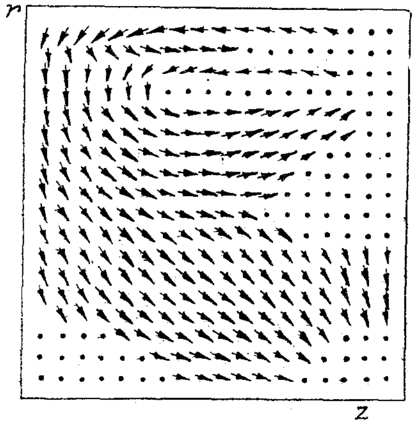

Fig. 2

The density distribution presented in Fig. 1 is chosen in accordance with experimental data [2] in the form

$$\rho(r, z, 0) = \begin{cases} \max(1; N\,\mathrm{ch}^{-2}\,\alpha(r - r_0)), & r < r_0, \\ \max(0, 1; N\,\mathrm{ch}^{-2}\,\alpha(r - r_0)), & r \geqslant r_0. \end{cases}$$

In Fig. 1 r_0 is the radius of the null surface and r_S is the radius of the surface behind which the plasma density is low. The initial gasdynamic pressure distribution of the plasma is determined from the condition of its equilibrium in the magnetic field.

The plasma is taken out of the equilibrium state through the assignment of a disturbance of the potential at the boundary of the calculated region at $r = R$: $A(R, z, t) = A_0(1 + A_1 \sin \omega t \exp(-\beta z^2))$. The constants A_1, ω, and β determine the amplitude, frequency, and "width" of the disturbance. The remaining boundaries of the region ($r = 0$, $z = z_1$, $z = 0$) are taken as lines or planes of symmetry. The chosen boundary conditions basically correspond to the data of [2].

The system of equations (2) with the indicated initial and boundary conditions was solved numerically by a scheme which is a natural generalization to the cylindrical case of the scheme presented in [5].

Let us examine the results of the numerical solution. Plasma motion develops under the action of the magnetic pressure which grows at the chamber wall. A clear concept of it can be obtained from Fig. 2, where the velocity fields at the time $t = 0.6$ are presented. Since the conductivity of the plasma is finite, reconnection of field lines takes place through the null surface of the magnetic field. The dynamics of this process essentially depends on the parameters of the plasma (conductivity, initial density drop) and of the outside disturbance (A_1, ω, β). A quantitative characteristic of the reconnection is the difference between the magnetic fluxes through the right ($z = z_1$) and left ($z = 0$) boundaries of the calculating region divided by the magnetic flux through the right boundary, i.e.,

$$\Pi(t) = (\Phi_1 - \Phi_0)/\Phi_1, \tag{3}$$

where

$$\Phi_0 = 2\pi \int_{r_{00}}^{r_{S0}} rH_z(r, 0, t)\,dr; \quad \Phi_1 = 2\pi \int_{r_{01}}^{r_{S1}} rH_z(r, z_1, t)\,dr.$$

Here r_{00} and r_{01} are the coordinates of the minimum value of the potential; r_{S0} and r_{S1} are the radii of the separatrix. The surface bounding a region within which the total magnetic flux equals zero is called a separatrix [2]. Values of r_{00}, r_{01}, r_{S0}, and r_{S1} are plotted in Fig. 3. We note that the values of r_{00} and r_{01} (r_{S0} and r_{S1}) at the boundaries $z = 0$ and z_1 do not coincide owing to the influence of the outside disturbance and conduction. The presence of a finite plasma conductivity results not only in reconnection but also in diffusion, a quantitative characterization of which can be obtained by analogy with Eq. (3) as the difference between the fluxes through the right boundary ($z = z_1$) at the times $t = 0$ and $t = t_1$:

$$\Pi_d(t_1) = \frac{\int_{r_{01}}^{r_{S1}} r\{H_z(r, z_1, 0) - H_z(r, z_1, t_1)\}\,dr}{\int_{r_{01}}^{r_{S1}} rH_z(r, z_1, 0)\,dr}.$$

TABLE 1

ν_0	10^{-3}	$2\cdot 10^{-3}$	$5\cdot 10^{-3}$	10^{-2}
$\Pi(t_*)$	0,15	0,35	0,40	0,89

TABLE 2

ν_1	10^{-3}	$2\cdot 10^{-3}$	$5\cdot 10^{-3}$	10^{-2}
$\Pi(t_*)$	0,1	0,23	0,3	0,68

TABLE 3

ν_0	10^{-3}	$2\cdot 10^{-3}$	$5\cdot 10^{-3}$	10^{-2}
$\Pi_d(t_*)$	0,28	0,75	0,82	0,91

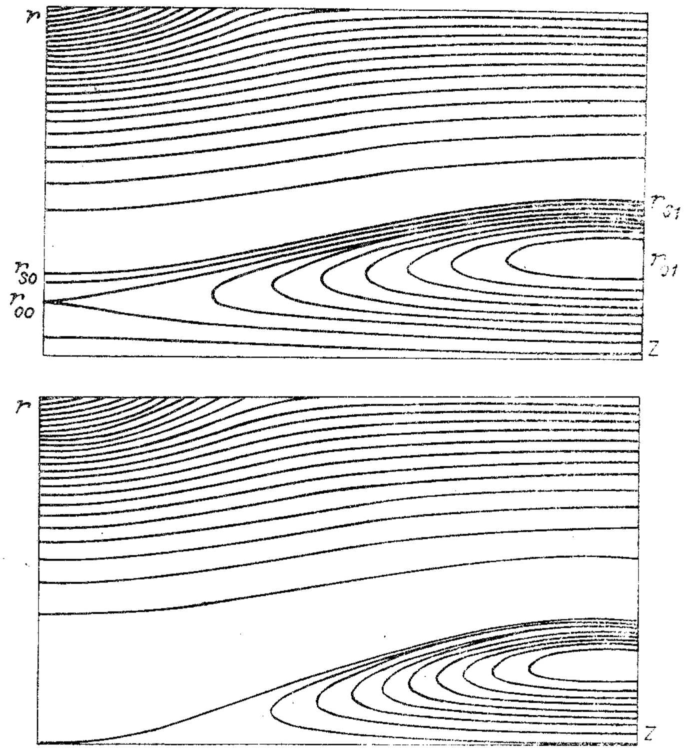

Fig. 3

In Table 1 we present the dependence of the quantity $\Pi(t_*)$ at a fixed time $t_* = 1.5$ on the anomalous collision frequency ν_0 for the following parameters of the plasma and the outside disturbance: $N = 3$, $A_1 = 0.2$, $\omega = 0.5$, $\beta = 50$, $r_0 = 0.5$, $\alpha = 15$.

In Table 2 we present the dependence of the quantity $\Pi(t_*)$ at a fixed time $t_* = 1.5$ on the Coulomb collision frequency $\nu = \nu_1 T^{-3/2}$ for the same parameters as in Table 1.

In Table 3 we present the dependence of the magnetic field diffusion $\Pi_d(t_*)$ on the anomalous collision frequency ν_0 for the same parameters. The reconnection $\Pi(t_*)$ depends on the width of the neutral sheet, i.e., on the parameter α, as follows: $\Pi(t_*) = 0.38$, 0.40, and 0.45 for $\alpha = 10$, 15, and 25 ($\delta = 0.4R$, 0.25R, and 0.15R). The reconnection $\Pi(t_*)$ also depends on the position of the neutral surface, i.e., on the parameter r_0, as follows: $\Pi(t_*) = 0.40$ and 0.31 for $r_0 = 0.5R$ and 0.35R.

The process under consideration is characterized not only by reconnection of field lines and magnetic field diffusion but also by the development of a compression wave and plasma motion along the cylinder axis. The plasma motion can be traced through an analysis of the variation of the mass of plasma, $M(z, t) = 2\pi \int_0^R r\rho(r, z, t)\, dr$, passing through a cross section of the cylindrical chamber. It follows from the calculated results that the compression wave propagates along the cylinder axis with a velocity $v = 1.2v_A$ ($v_A = H_0 \cdot (4\pi\rho_0)^{-1/2}$).

Magnetic field lines at the successive times $t = 1.0$ and 1.5 ($\alpha = 15$, $\omega = 0.5$, $A_1 = 0.2$, $\nu_1 = 0.005$, $\nu_0 = 0$) are presented in Fig. 3. The dynamics of the process of magnetic field line reconnection and the formation of a closed magnetic field configuration are seen from these graphs.

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A SEMI-SELF-MAINTAINED VOLUMETRIC DISCHARGE IN ITS OWN MAGNETIC FIELD

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A volumetric discharge excited in a gas by an electron beam has found wide application in electric-ionization lasers [1]. A number of reports have recently appeared in which the influence of the magnetic field of the discharge current and the electron beam on the uniformity of such a discharge is investigated [2-9].

If the discharge power is expressed through the electric field strength and the Larmor radius of the electron beam [4], then the radiant energy Q taken from a unit length of a laser pulse in one pulse can be written in the form

$$Q = 2\eta \frac{1}{e} \frac{E}{p} p t_{pu} \sqrt{U_b (U_b + 2mc^2)} \frac{d}{r_L} \frac{1}{\eta_0},$$

where E is the electric field strength; d, distance between electrodes; p, pressure of the laser mixture; U_b, energy of the beam electrons; t_{pu}, time of pumping of the active medium of the laser; η, efficiency of conversion of electrical energy into radiant energy; $\eta_0 = (\mu_0/\varepsilon_0)^{1/2}$; μ_0 and ε_0, magnetic permeability and the permittivity of a vacuum; r_L, minimum Larmor radius of beam electrons with an energy U_b in the magnetic field of the discharge current. Means of increasing the radiant energy are obvious from the expression for Q: an increase in the values of the parameters E/p, pt_{pu}, d/r_L, and U_b for the optimum η. Realistic possibilities for varying these parameters have essential limits, however: E/p is limited to the vicinity of values where the pumping of laser levels is efficient and the quantity pt_{pu} is limited by gas heating and by relaxation of the upper laser level [1, 10]; the use of high-energy beams with an energy of 0.5-1 MeV requires special technical equipment and considerably reduces the efficiency of the entire laser system. Therefore, d/r_L is the only free parameter permitting an increase in the radiant energy. Consequently, a detailed investigation of the uniformity of a semi-self-maintained discharge in its own magnetic field must precede the creation of superpowerful laser systems.

The influence of a magnetic field on the distribution of beam ionization losses was first studied theoretically in [2, 3] by the Monte Carlo method. The self-consistent problem of the uniformity of a discharge in its own magnetic field was analyzed in [6], a nonsteady solution without allowance for scattering of beam electrons was obtained in [8], and a model kinetic equation for beam electrons was investigated in [9]. In [5-7] it was shown that for a given magnetic field there exists a limiting beam width, and an increase in beam width beyond it does not result in a change in the active region of the discharge. The existence of an optimum magnetic field providing the best uniformity of ionization losses of the beam and the promise of the use of relatively narrow electron beams (h < d, where h is beam width up to the foil) to create superpowerful laser systems are dis-

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